UNIVERSITÄT SIEGEN

FACHBEREICH ELEKTROTECHNIK UND INFORMATIK

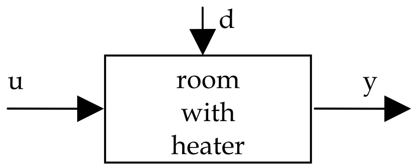
Institut für Regelungs- und Steuerungstechnik   Prof. Dr.-Ing. R. Mayr

Basic Control

1. Exercise

1. Task:

A heater has to be extended by a thermostatic control systen. First the function of the heater must be examined.



a) The simplified flow diagram looks as follows:
   u: Combustible advection
   y: Inside temperature
   d: Outside temperature
   Is it a feedback control system here?

b) The handwheel for combustible advection is calibrated with regard to an outside temperature of 10° C. The interior room temperature is 15° C and by adjusting the handwheel to the 18° C tag it must be increased. Which input function is present at t=0 [s]? Give a qualitative temperature trajectory for the interior room at outside temperatures of 5° C, 10° C and 12° C.

c) Extend the block diagram listed above by the corresponding elements so that a temperature feedback control system results. Which kind of non-linear automatic controller do you recommend?

d) Draw the qualitative temperature trajectory in the case of an increase of the temperature of 15° C to 18° C at an outside temperature of 5° C. Describe the advantage of this kind of feedback control system.

e) The heater can be switched to a high power, where the combustible advection is duplicated. Draw for this in accordance with subtask d) the qualitative trajectory.

2. Task:

Outline the corresponding flow diagrams for these differential equations:

a) $\ddot{x}(t) + c_1 x(t) - 17 = \sqrt{c_1}\,\ddot{x}(t)$

b) $\ddot{x}(t) + c_1 \sqrt{x(t)} - 3 = 0$

c) $\ddot{x}(t) - c_1 x(t - c_2) = \sin(c_1)$

d) $\ddot{x}(t) + c_1 \dot{x}(t)\, x(t) = 1$

e) $\frac{1}{x(t)} + \ddot{x}(t) = x(t)$

Do these differential equations have linear character? Give in each case an explanation.

f) Perform this analysis also for the subsequently shown system of differential equations:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_3(t)$$
$$\dot{x}_3(t) = c_3 x_3(t) + c_2 x_2(t) + c_1 x_1(t) + c_4 c_5$$

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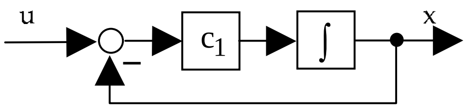
<u>Basic Control</u>

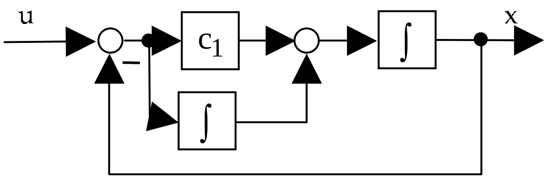
2. Exercise

<u>1. Task:</u>
The following flow diagrams have to be analysed and to be transformed into corresponding differential equations:

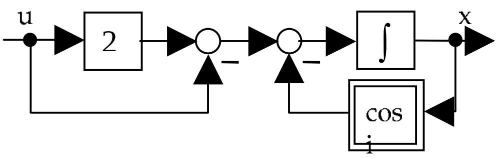
a)



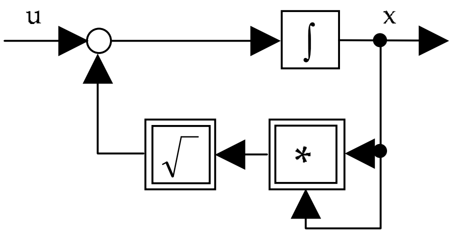
b)



c)



d)

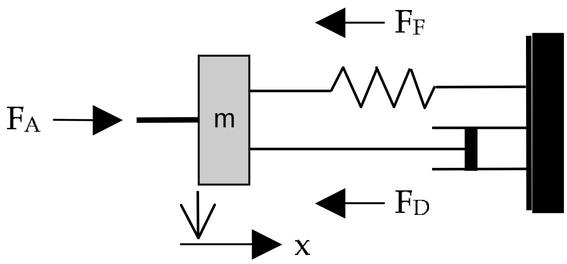


<u>2. Task:</u>
A technical system consisting of a mass m, of a spring and a damper exists here.
On this system the force F_A is acting. The shift of the mass is *x*. The idle condition of the systems is defined by the state $x = 0$ as well as all derivatives $\dot{x} = \ddot{x} = ... = 0$ and $F_A = 0$.

Furthermore, the following is valid:

Spring equation: $F_F(t) = c\,x(t)$

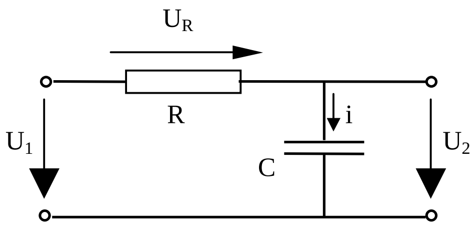
Damper equation: $F_D(t) = d\,\dot{x}(t)$



a) Indicate the differential equation for this system. Of which order is this differential equation? Is it a linear differential equation?
b) Set up the corresponding flow diagram for that spring damper system.

3. Task:

Given is a network consisting of a resistor R and a capacity C. The input voltage is U_1.



a) Determine the differential equation, by which the network can be described mathematically.
Of which order is the system?

b) Designate the class to which this system appertains. Outline the flow diagram.

c) Calculate and draw the trajectory of the output voltage U_2 in case that the input voltage U_1 describes a step function from 0V to 2V.

d) How would the trajectory be in case that the resistor R would be twice as high and the capacity C would only be half of as high?

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3. Exercise

1. Task:

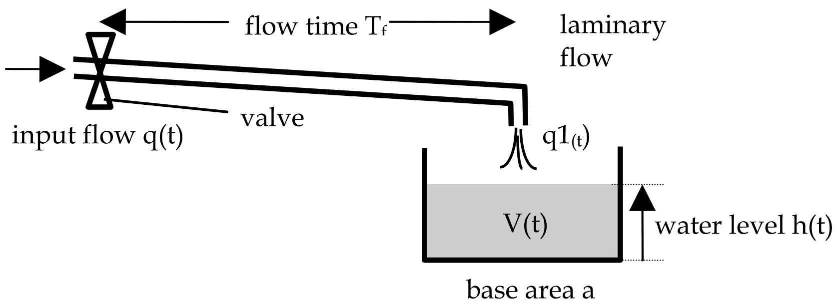
Given is a function existing of 2 δ-impulses

$$w(t) = \delta(t) - \delta(t-2) .$$

a) Draw this function.
b) Calculate and outline the result of the integral of this function.
c) Furthermore, calculate and outline the result of the integral of the result from subtask b).

2. Task:

For a water basin with input flow the complex transfer function has to be determined.



a) Determine the complex transfer function $F(s) = \frac{H(s)}{Q(s)}$. The initial values are set to zero.
b) Outline the corresponding flow diagram.
c) Of which class is the system?

3. Task:

Given is the system

$$\ddot{y}(t) + b_1 \dot{y}(t) + b_o y(t) = a_o u(t) + a_1 \dot{u}(t)$$

a) Determine the complex transfer function corresponding to this differential equation. The initial values are set to zero.
b) On the input the impulse function $u(t) = \delta(t)$ is acting. Based on the initial and final value theorems of the Laplace transformation calculate the values at the system input and output for $t \to O^+$ and $t \to \infty$.
c) Perform the same calculations as in subtask b), but with the difference, that on the system input now the step function $u(t) = \sigma(t)$ is acting.
d) Carry out again the same calculations as in the subtasks b) and c), but with the difference that now on the system input the ramp function $u(t) = t$ is acting.
e) Compare the results from the subtasks b), c) and d) and give appropriate explanations.

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4. Exercise

<u>1. Task:</u>

Given is the output variable Y(s) in the frequency domain:

$$Y(s) = \frac{2s^2 + 3s + 2}{s^3 + 3s^2 + 5s + 3}$$

a) Perform for Y(s) the partial fraction decomposition

Reference: $s^3 + 3s^2 + 5s + 3 = (s+1)\,(s^2+2s+3)$

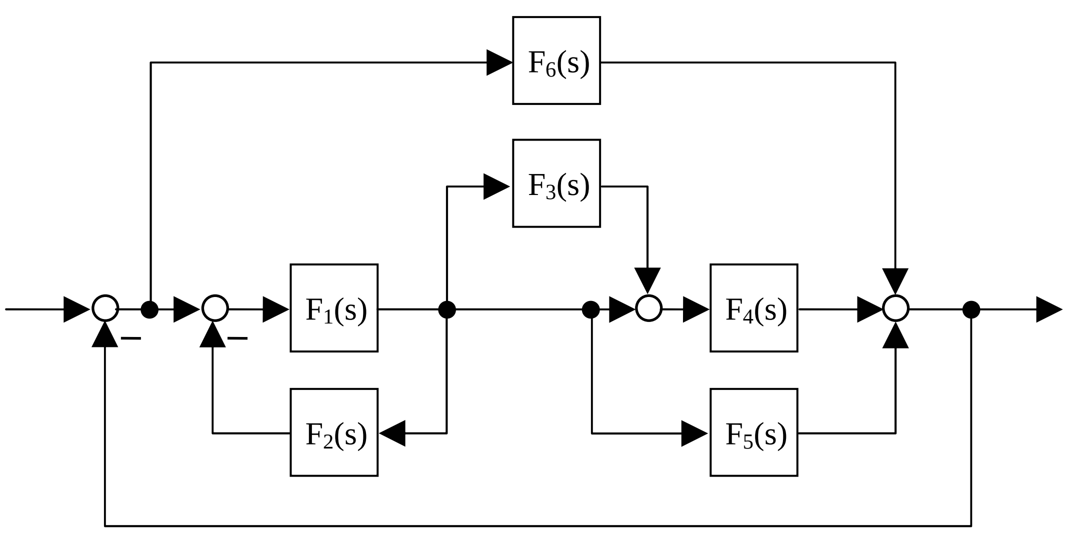
b) Carry out the inverse Laplace transformation by using the corresponding lookup table.

c) Draw the corresponding function y(t) in the time domain.

d) In case that as input variable the impulse function $u(t) = \delta(t)$ is acting, give an approximation about the dynamic behaviour of the underlying system.

<u>2. Task:</u>

Simplify the following flow diagram:



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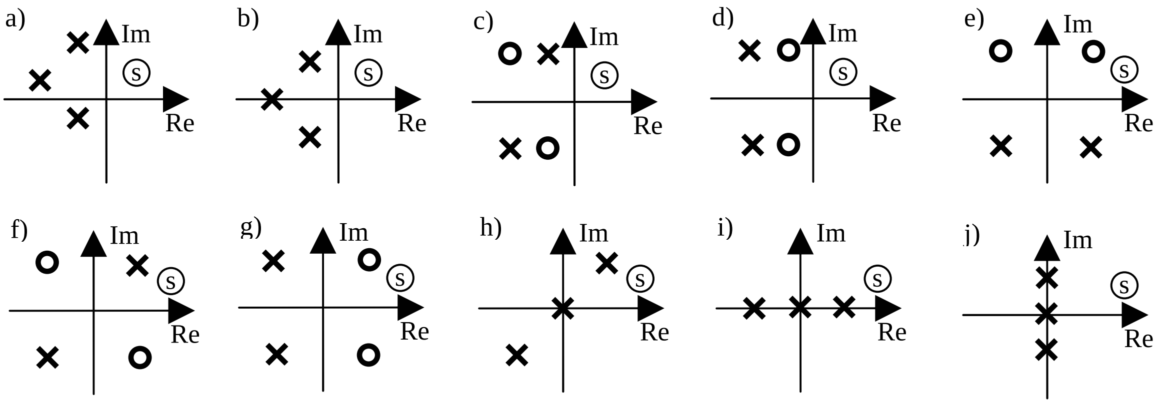
5. Exercise

1. Task:

   Indicate the pole-/zero distributions for the following systems:
   a) Stable system of 1st order
   b) Unstable system of 1st order (Can this system perform periodic oscillations by itself?)
   c) Stable aperiodic system of 2nd of order
   d) Aperiodic borderline case is (stable)
   e) Periodic stable system of 2nd order

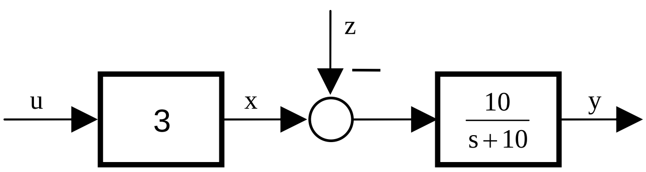
2. Task:

   Can the following systems occur due to linear control theory?
   Which cases of controlled systems are realized in each case?



3. Task:

   A motor, which can be approximated by a PT_1-element, is given:



Here are:

u(t)	control signal (input voltage)
x(t)	intermediate signal (power of engine)
y(t)	output signal (revolutions per minute)
z(t)	disturbance (opposite force)

a) Develop a P-controller, which moves the pole for the overall system (closed loop) to $s_1 = -70$.
   Calculate the steady state control deviation as well as the disturbance transfer function.
b) Develop a PI-controller with the poles at $s_1 = -60$ and $s_2 = -70$ for the overall system.
   Calculate also here the steady state control deviation as well as the disturbance transfer function.

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6. Exercise

1. Task:

The following controlled system is given with the complex transfer function:

$$F_s(s) = \frac{1}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{s+2}$$

This controlled system has to be equipped with a P-controller, which has the control gain K_R.

a) Determine the nominal value transfer function $F_w(s)$ the overall system (closed loop).

b) Evaluate the characteristic equations for both cases considering $K_R = 5$ and $K_R = 10$ for the overall system on.

c) Set up the determinants relevant to the Hurwitz criterion.

c) Determine the range of values for the control gain K_R , where the closed loop is stable.

2. Task:

The controlled system

$$F_S(s) = \frac{1}{s^3 + 3s^2 + 2s} ,$$

has to be extended by a P-controller.
This control system must be examined based on the root locus method.

a) Calculate and construct the root locus by the application of the corresponding rules.

b) Indicate the value for the stability borderline, where the root locus goes through the imaginary axis.

3. Task:

The controlled system of 2nd order

$$F_s(s) = \frac{1}{(s+1)(s+3)}$$

shall be controlled by the means of a P-controller. Construct the corresponding root locus.
Evaluate the value for the control gain regarding the aperiodic borderline case.

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7. Exercise

1. Task:

A controlled system of 2nd order with 2 poles and one zero is given:

$$F(s) = K_s \frac{s - s_{N1}}{(s - s_{P1})(s - s_{P2})}$$

The amplification factor is set to $K_S = 1$.

a) Determine the root locus for $s_{N1} = -1$, $s_{P1} = -2$ and $s_{P2} = -3$.

b) Further, determine the root locus for $s_{N1} = -3$, $s_{P1} = -1$ and $s_{P2} = -2$.

Reference: Neighbouring curve pieces of the root locus on the real axis in the complex s-plane can be connected by arches, and in simpler cases as in the present task by arcs of a circle.

2. Task:

The controlled system with the complex transfer function

$$F_S(s) = \frac{1}{s^3 + 3s^2 + 2s}$$

has to be extended by a P-controller with the control gain K_R.

The calculations shall be carried out with the values $K_R = 4$ and $K_R = 8$ for the control gain.

a) Determine the real and the imaginary parts of $F_o(j\omega)$ generally.

b) Compute the values of the real and imaginary parts of $F_o(j\omega)$ considering both values for K_R as well as the frequencies: $\omega = 0.01, 0.1, 1.0, 1.2, 1.5$ and 10.

c) Outline both locus curves in the F_o-plane and mark the point -1.

d) Determine stability for both locus curves by the application of the general valid Nyquist criterion.

e) May the simplified Nyquist criterion be applied here?

f) Determine the stability again for both locus curves by the application of the simplified Nyquist criterion.